Boneh-Franklin Identity Based Encryption Revisited

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Outline

- Definitions
  - Identity Based Encryption (IBE)
  - Secure IBEs
  - Bilinear maps and problems

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  - Bilinear maps and problems
- Boneh-Franklin IBE scheme revisited
  - A flaw in the security reduction
  - New security proof

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Decrypt Takes as inputs params, private key d_{id} and message C and outputs a message m.

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Guess $\mathcal{A}$ outputs a bit β' and wins if $\beta' = \beta$.

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- $e(aP, bP) = e(P, abP) = e(abP, P) = e(P, aP)^b = e(P, P)^{ab}$
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Bilinear Diffie-Hellman (BDH) Problem on $\mathbb{G}$. Given $P, aP, bP, cP \in \mathbb{G}$ as input, where $a, b, c \leftarrow \mathbb{Z}_p$ compute $\widehat{e}(P, P)^{abc} \in \mathbb{G}_1$.

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Decision Bilinear Diffie-Hellman (DBDH) Problem on $\mathbb{G}$. Given $P, aP, bP, cP \leftarrow \mathbb{G}$, where $a, b, c \leftarrow \mathbb{Z}_p$, and $T \leftarrow \mathbb{G}_1$, as inputs; tell apart $(P, aP, bP, cP, \widehat{e}(P, P)^{abc})$ from (P, aP, bP, cP, T).

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In the Random Oracle Model $q_{\mathsf{RO}} \leq 2^t$, and it is often assumed that $q_D, q_E \leq 2^{30}$.

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- params $= \langle P, P_{pub}, H_1, H_2, H_3, H_4 \rangle$.
- The master-key is $s \in \mathbb{Z}_p^*$.

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Decrypt. To decrypt $C = \langle U, V, W \rangle \in \mathbb{G} \times \{0,1\}^n \times \{0,1\}^n$

- Compute $V \oplus H_2(\widehat{e}(U, d_{\mathsf{id}})) = \sigma$ and $W \oplus H_4(\sigma) = M$.
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$$\widehat{e}(U, d_{\mathsf{id}}) = \widehat{e}(rP, sQ_{\mathsf{id}}) = \widehat{e}(sP, rQ_{\mathsf{id}}) = \widehat{e}(P_{pub}, Q_{\mathsf{id}})^r = g_{\mathsf{id}}^r$$

Security intuition/statement

Sender	aP, bP, c	$\rightarrow$	$\widehat{e}(P,P)^{abc} = \widehat{e}(aP, bP)^{c}$
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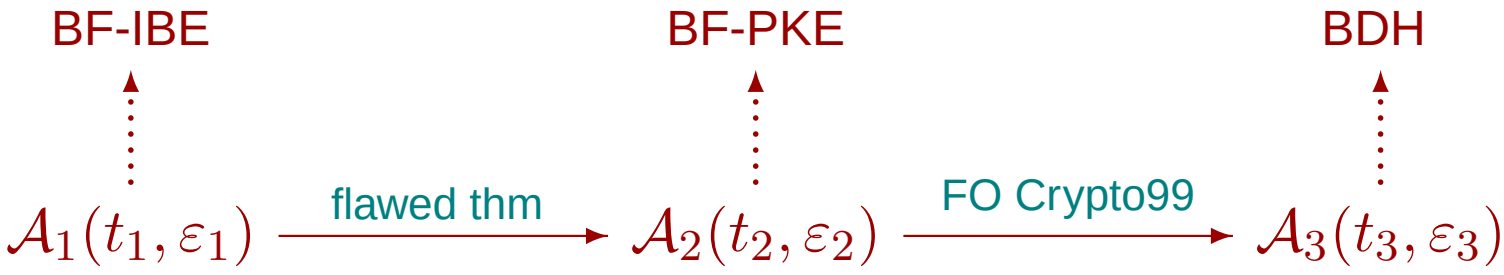
Theorem Let $\mathcal{A}$ an IND-ID-CCA adversary running in time t and with advantage ε against BF-IBE making at most q_E, q_D, q_H queries. Then there exists $\mathcal{B}$ running in time roughly t with advantage at least $\frac{\varepsilon}{q_H^2 q_D}$ against BDH problem in $\mathbb{G}$.

Security reduction strategy



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Theorem Let $\mathcal{A}$ an IND-ID-CCA adversary that has advantage ε against BF-IBE making at most q_H, q_E, q_D queries. Then there is an IND-CCA adversary $\mathcal{B}$ that has advantage at least $\frac{\varepsilon}{(1+q_E+q_D)}$ against BF-PKE.

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But the checking $H_3(\sigma, m) = b^{-1}rP$ does (almost) never hold!.

BF security reduction is not valid!

Fixing the reduction

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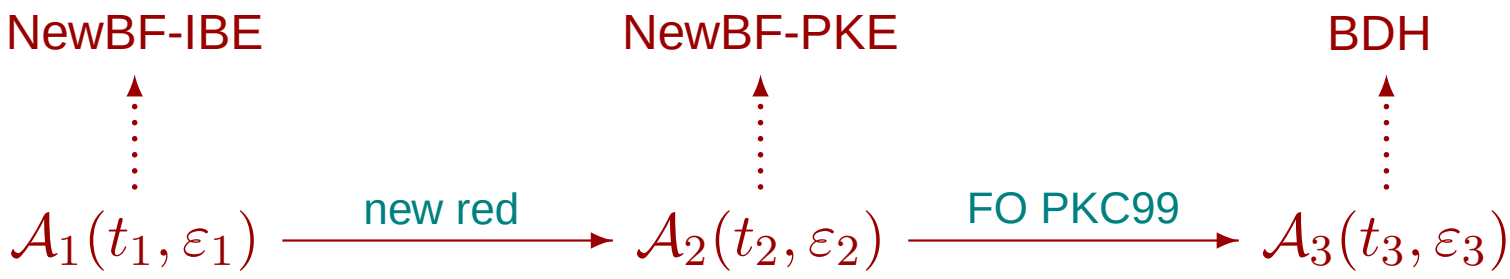
Worse reduction :-(

IBE scheme with tighter reduction



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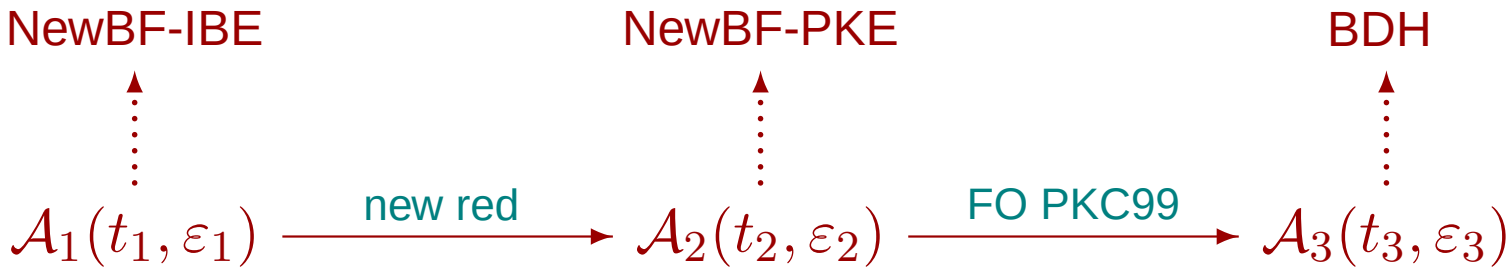

Theorem Let $\mathcal{A}(t, \varepsilon)$ breaking IND-ID-CCA security of NewBF-IBE making at most q_H queries. Then there exists $\mathcal{B}(t, \frac{\varepsilon}{q_H^2})$ against BDH problem in $\mathbb{G}$.

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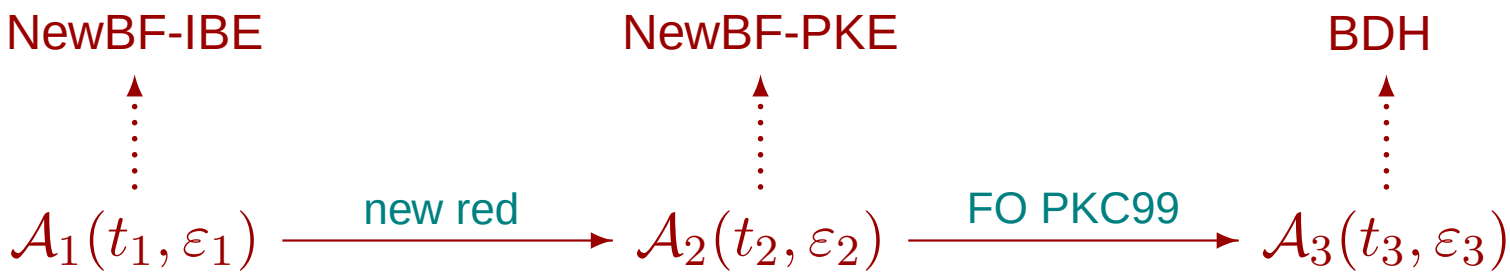

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The new scheme has shorter and more compact ciphertexts, uses one less random oracle, has a tighter reduction ($\frac{\varepsilon}{q_H}$ with respect to decisional BDH.)

Conclusions

- A flaw in BF security reduction has been pointed out and solved.
- A New-BF scheme with better performance and security properties.
- These remarks can be applied to a considerable number of works in the literature.

This is the end...